THE EFFECT OF EARLY STOPPING

MICHEL VAN DEN BERGH

1. Preliminaries

If we do an SPRT for[1] H0:$\mu = \mu_0$ versus H1:$\mu = \mu_1$ with LLR bounds $a < 0 < b$ then by [2] the pass probability is approximately

$$p_{\text{pass}} = \frac{1 - e^{-ha}}{e^{-hb} - e^{-ha}}$$

where

$$h = \frac{2\mu - (\mu_0 + \mu_1)}{\mu_1 - \mu_0}$$

In other words: h is the relative position of the true μ in the interval $[\mu_0, \mu_1]$ with $h = -1, +1$ corresponding respectively to $\mu = \mu_0, \mu_1$.

2. Missed passers

Assume $h = 1$ (i.e. H1 holds) and $x \in [a, 0[$. Our aim is to compute the probability that the test touches LLR $= x$ and then still passes. One may view this as the probability of a failing test with bounds $[x, b]$ combined with a passing test with bounds $[a - x, b - x]$.

Thus the sought probability is

$$\left(1 - \frac{1 - e^{-x}}{e^{-b} - e^{-x}}\right) \frac{1 - e^{-(a-x)}}{e^{-(b-x)} - e^{-(a-x)}} = \frac{e^{-b} - 1}{e^{-b} - e^{-x}} \frac{e^{-x} - e^{-a}}{e^{-b} - e^{-a}}$$

Assuming error probabilities $\alpha = \beta = 0.05$ so that $a = -\log(19)$, $b = \log(19)$, then the probability of a missed passer is as in Figure 1. We see that ∼30% of the passers will hit LLR $= -1.0$ and ∼8% of the passers will even hit LLR $= -2.0$.

3. Error probabilities

If we stop at LLR $= x$ then the error probabilities are respectively

$$\alpha = \frac{1 - e^{x}}{e^{b} - e^{x}}$$

$$\beta = 1 - \frac{1 - e^{-x}}{e^{-b} - e^{-x}}$$

Under the assumption that the design values of α and β are equal to 0.05 this yields Figure 2.

[1] μ is typically the match score but in fact it can be any parameter according to [1].

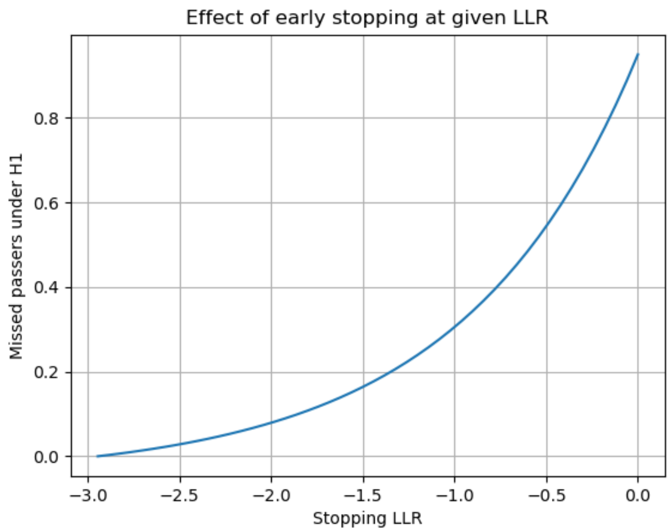

FIGURE 1. Missed passers

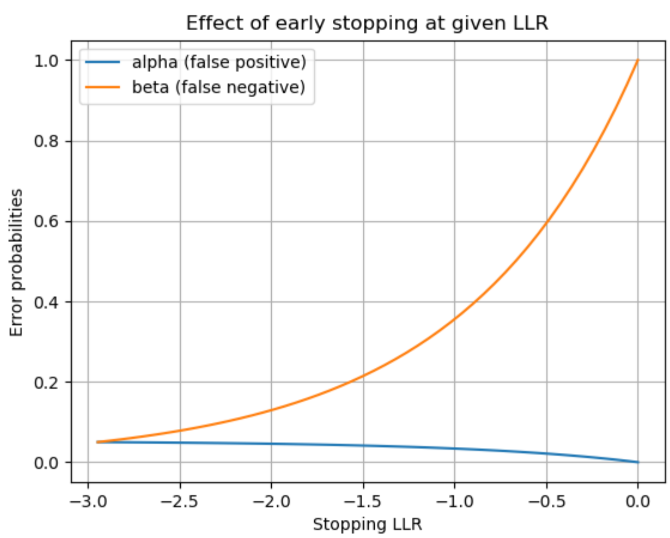

FIGURE 2. Error probabilities

REFERENCES

[1] Michel Van den Bergh. A practical introduction to the GSPRT. `http://www.cantate.be/Fishtest/GSPRT_approximation.pdf`.

[2] Michel Van den Bergh. The SPRT for Brownian motion. `http://www.cantate.be/Fishtest/sprta.pdf`.